

# Spectrum of single-photon emission and scattering in cavity optomechanics

Jie-Qiao Liao, H. K. Cheung, and C. K. Law

*Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong Special Administrative Region, People's Republic of China*

(Dated: February 29, 2012)

We present an analytic solution describing the quantum state of a single photon after interacting with a moving mirror in a cavity. This includes situations when the photon is initially stored in a cavity mode as well as when the photon is injected into the cavity. In addition, we obtain the spectrum of the output photon in the resolved-sideband limit, which reveals spectral features of the single-photon strong-coupling regime in this system. We also clarify the conditions under which the phonon sidebands are visible and the photon-state frequency shift can be resolved.

PACS numbers: 42.50.Wk, 42.50.Pq, 07.10.Cm

Cavity optomechanics [1, 2] is a new frontier for exploring the coherent coupling via radiation pressure between electromagnetic and mechanical degrees of freedom. Several recent experimental systems in cavity optomechanics [3–5] approached the single-photon strong-coupling regime, in which the radiation pressure from a single photon can produce observable effects. Such a strong coupling is important to realize various proposals of generating macroscopic superposition states for testing quantum theory [6–8]. In parallel with the progress in experiments, some theoretical investigations have also begun [9–11] to explore the effects of radiation-pressure coupling at few- and even single-photon levels. For example, Rabl [9], and Nunnenkamp and coworkers [10] have independently studied the statistics of cavity photons in the single-photon strong-coupling regime when the cavity is weakly driven by a continuous-wave laser.

Physically, in the single-photon strong-coupling regime, the mirror's displacement induced by a single photon can significantly affect the cavity field, leading to some observable features in the spectrum of single-photon emission and scattering. Thus, a natural question is how these spectra may characterize the single-photon strong-coupling regime. In this Brief Report we answer the question by calculating analytically the spectrum of single-photon emission and scattering in a cavity optomechanical system. In particular, we indicate the relation between the photon spectral characteristic and the interaction strength  $g$  [cf. Eq. (1)] of radiation pressure per photon.

To begin with, we specify the system of a Fabry-Pérot-type optomechanical cavity formed by a fixed end mirror and a moving end mirror [Fig. 1(a)]. The cavity field and the mirror motion are coupled to each other via radiation pressure. The Hamiltonian of the optomechanical cavity is

$$H_{\text{opc}} = \hbar\omega_c a^\dagger a + \hbar\omega_M b^\dagger b - \hbar g a^\dagger a (b^\dagger + b), \quad (1)$$

where  $a$  ( $a^\dagger$ ) and  $b$  ( $b^\dagger$ ) are, respectively, the annihilation (creation) operators of the electromagnetic and mechanical modes, with the respective resonant frequencies  $\omega_c$  and  $\omega_M$ . The parameter  $g = \omega_c x_{\text{zpf}}/L$  is the single-photon coupling strength of the radiation pres-

sure between the cavity and the mirror, where  $x_{\text{zpf}} = \sqrt{\hbar/(2M\omega_M)}$  is the zero-point fluctuation of the mirror with mass  $M$ , and  $L$  is the rest length of the cavity.

Let us denote the harmonic-oscillator number states for the cavity and the mirror as  $|m\rangle_a$  and  $|n\rangle_b$  ( $m, n = 0, 1, 2, \dots$ ), respectively, then the eigensystem of Hamiltonian (1) can be written as

$$H_{\text{opc}}|m\rangle_a|\tilde{n}(m)\rangle_b = \hbar(m\omega_c + n\omega_M - m^2\delta)|m\rangle_a|\tilde{n}(m)\rangle_b \quad (2)$$

for  $n, m = 0, 1, 2, \dots$ , where  $\delta = g^2/\omega_M$  is the photon-state frequency shift caused by a single-photon radiation pressure. The state

$$|\tilde{n}(m)\rangle_b \equiv \exp[m\beta_0(b^\dagger - b)]|n\rangle_b \quad (3)$$

is an  $m$ -photon displaced number state [12] with  $\beta_0 = g/\omega_M$ . Particularly,  $|\tilde{n}(1)\rangle_b$  is the single-photon displaced number state; and  $|\tilde{n}(0)\rangle_b$  is the zero-photon displaced number state, which is the same as the harmonic-oscillator number state  $|n\rangle_b$  by Eq. (3). For convenience, the energy-level structure of the optomechanical cavity in the zero- and one-photon cases is shown in Fig. 1(b).

In optomechanical systems, the decay rate  $\gamma_m$  of the mirror motion can be much smaller than the cavity-field's decay rate  $\gamma_c$ , then during the time interval  $1/\gamma_c \ll t \ll 1/\gamma_m$ , the damping of the mirror motion is negligible. In the following we merely consider the dissipation of the cavity field by coupling it to continuous outside fields so that the full Hamiltonian is

$$H = H_{\text{opc}} + \int_0^\infty \hbar\omega_k c_k^\dagger c_k dk + \hbar\xi \int_0^\infty (c_k^\dagger a + a^\dagger c_k) dk, \quad (4)$$

where  $c_k$  ( $c_k^\dagger$ ) is the annihilation (creation) operator of the  $k$ th outside-field mode with frequency  $\omega_k$ , and  $\xi$  is the photon-hopping interaction strength.

Denoting  $|\tilde{n}(1)\rangle_b = |\tilde{n}\rangle_b$  for concision, a general state in the single-photon subspace of the total system can be expressed as

$$|\varphi(t)\rangle = \sum_{n=0}^{\infty} A_n(t) |1\rangle_a |\tilde{n}\rangle_b |\emptyset\rangle + \sum_{n=0}^{\infty} \int_0^\infty B_{n,k}(t) |0\rangle_a |n\rangle_b |1_k\rangle dk, \quad (5)$$

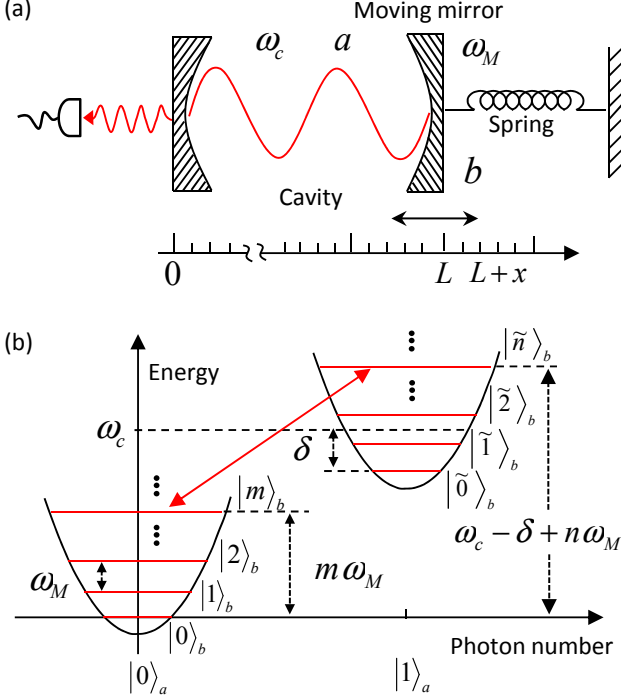


FIG. 1: (Color online) (a) Schematic diagram of a Fabry-Perot-type optomechanical cavity formed by a fixed end mirror and a moving end mirror. (b) The energy-level structure of the optomechanical cavity (limited in the zero- and one-photon subspaces). Here we denote  $|\tilde{n}(0)\rangle_b = |n\rangle_b$  and  $|\tilde{n}(1)\rangle_b = |\tilde{n}\rangle_b$ .

where  $|1\rangle_a|\tilde{n}\rangle_b|\emptyset\rangle$  stands for the state with one photon in the cavity, no photon in these outside fields, and the mirror in displaced number state  $|\tilde{n}\rangle_b$  (hereafter we call the single-photon displaced number state as displaced number state when there is no confusion), while  $|0\rangle_a|n\rangle_b|1_k\rangle$  denotes the state with a vacuum cavity field, one photon in the  $k$ th mode of these continuous fields, and the mirror in number state  $|n\rangle_b$ . In Eq. (5), we have used the completeness relation of these displaced number states  $|\tilde{n}\rangle_b$  ( $n = 0, 1, 2, \dots$ ). According to the Schrödinger equation, we can obtain equations of motion [in the rotating picture with respect to  $\hbar\omega_c(a^\dagger a + \int_0^\infty c_k^\dagger c_k dk)$ ] for probability amplitudes as

$$\begin{aligned} \dot{A}_m(t) = & -i(m\omega_M - \delta)A_m(t) \\ & -i\xi \sum_{n=0}^{\infty} \int_0^\infty \langle \tilde{m}|_b|n\rangle_b B_{n,k}(t) dk, \end{aligned} \quad (6a)$$

$$\begin{aligned} \dot{B}_{m,k}(t) = & -i(m\omega_M + \Delta_k)B_{m,k}(t) \\ & -i\xi \sum_{n=0}^{\infty} \langle m|_b|\tilde{n}\rangle_b A_n(t), \end{aligned} \quad (6b)$$

where  $\Delta_k = \omega_k - \omega_c$  is the detuning of the  $k$ th mode photon from the cavity frequency. In the following we discuss the long-time solution and the spectrum of two

cases.

*Single-photon emission.* In this case, we assume that a single photon is initially stored in the cavity mode, and such a photon will be transmitted (or emitted) out of the cavity. We first calculate the case where the initial state of the mirror is harmonic-oscillator number state  $|n_0\rangle_b$ . Once the solution for this case is found, the solution for general initial states of the mirror can be obtained accordingly by superposition. For the initial state  $|\varphi(0)\rangle = |1\rangle_a|n_0\rangle_b|\emptyset\rangle$ , we have

$$A_m(0) = \langle \tilde{m}|_b|n_0\rangle_b, \quad B_{m,k}(0) = 0. \quad (7)$$

The transient solution of Eq. (6) can be obtained by the Laplace transform method. The results are

$$A_{n_0,m}(t) = \langle \tilde{m}|_b|n_0\rangle_b e^{-[\frac{\gamma_c}{2} + i(m\omega_M - \delta)]t}, \quad (8a)$$

$$\begin{aligned} B_{n_0,m,k}(t) = & \sqrt{\frac{\gamma_c}{2\pi}} \sum_{n=0}^{\infty} \frac{\langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b}{\Delta_k + \delta - (n-m)\omega_M + i\frac{\gamma_c}{2}} \\ & \times \left( e^{-i(m\omega_M + \Delta_k)t} - e^{-[\frac{\gamma_c}{2} + i(n\omega_M - \delta)]t} \right), \end{aligned} \quad (8b)$$

where  $\gamma_c = 2\pi\xi^2$  is the cavity-field decay rate. Notice that the subscript  $n_0$  in  $A_{n_0,m}(t)$  and  $B_{n_0,m,k}(t)$  is added to mark the mirror's initial state  $|n_0\rangle_b$ . In the long-time limit, we have  $A_{n_0,m}(\infty) = 0$  and

$$B_{n_0,m,k}(\infty) = \sqrt{\frac{\gamma_c}{2\pi}} \sum_{n=0}^{\infty} \frac{\langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b e^{-i(m\omega_M + \Delta_k)t}}{\Delta_k + \delta - (n-m)\omega_M + i\frac{\gamma_c}{2}}. \quad (9)$$

We point out that the long-time limit here actually refers to the time scale of  $1/\gamma_c \ll t \ll 1/\gamma_m$ . During this time duration, the single photon leaks completely out of the cavity and the damping of the mechanical motion is negligible.

To understand Eq. (9), we note that the mirror's initial state  $|n_0\rangle_b$  is a superposition of displaced number states  $|n_0\rangle_b = \sum_{\tilde{n}=0}^{\infty} \langle \tilde{n}|_b|n_0\rangle_b |\tilde{n}\rangle_b$ . The transition  $|\tilde{n}\rangle_b \rightarrow |m\rangle_b$  can occur in the process of cavity photon leakage  $|1\rangle_a \rightarrow |0\rangle_a$ . For the process  $|1\rangle_a|\tilde{n}\rangle_b \rightarrow |0\rangle_a|m\rangle_b$ , the frequency of the emitted photon is governed by the resonance condition [see Fig. 1(b)]

$$\Delta_k = (n-m)\omega_M - \delta, \quad (10)$$

which is consistent with the real part of the pole in Eq. (9), [i.e.,  $\Delta_k + \delta - (n-m)\omega_M = 0$ ]. The amplitude for the process is proportional to the overlap  $\langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b$ , which can be calculated with the relation [12]

$$\langle m|_b|\tilde{n}\rangle_b = \begin{cases} \sqrt{\frac{m!}{n!}} e^{-\frac{\beta_0^2}{2}} (-\beta_0)^{n-m} L_m^{n-m}(\beta_0^2), & n \geq m, \\ \sqrt{\frac{n!}{m!}} e^{-\frac{\beta_0^2}{2}} \beta_0^{m-n} L_n^{m-n}(\beta_0^2), & m > n, \end{cases} \quad (11)$$

where  $L_r^s(x)$  is the associated Laguerre polynomial.

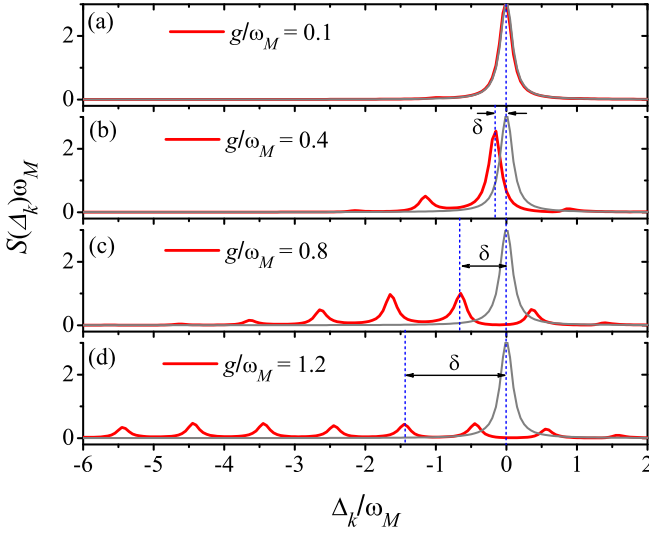


FIG. 2: (Color online) Spectrum  $S(\Delta_k)$  (red curves) of single-photon emission versus  $\Delta_k$  for various  $g$ . The mirror's initial state is  $|0\rangle_b$  and  $\gamma_c/\omega_M = 0.2$ . The dark gray curve is a Lorentzian spectrum for comparison.

A useful quantity in this system is the final reservoir occupation spectrum  $S(\Delta_k)$  [13], which is the probability density for finding the single photon in the  $k$ th mode of these outside fields. When the mirror is initially in a pure state  $\sum_{n_0=0}^{\infty} C_{n_0} |n_0\rangle_b$  or a mixed state  $\sum_{n_0=0}^{\infty} P_{n_0} |n_0\rangle_b \langle n_0|_b$ , the single-photon emission spectra are, respectively, given by

$$S(\Delta_k) = \sum_{m=0}^{\infty} \left| \sum_{n_0=0}^{\infty} C_{n_0} B_{n_0, m, k}(\infty) \right|^2, \quad (12a)$$

$$S(\Delta_k) = \sum_{m=0}^{\infty} \sum_{n_0=0}^{\infty} P_{n_0} |B_{n_0, m, k}(\infty)|^2. \quad (12b)$$

For a thermal equilibrium state  $\rho_b^{th}$  with thermal phonon number  $\bar{n}_b$ , we have  $P_{n_0} = \bar{n}_b^{n_0} / (\bar{n}_b + 1)^{n_0+1}$ .

To observe the spectral signatures of the coupling strength  $g$ , in Fig. 2 we plot the spectrum  $S(\Delta_k)$  of single-photon emission as a function of the photon frequency  $\Delta_k$  (in rotating picture) for various values of  $g$  when the mirror is initially prepared in ground state  $|0\rangle_b$ . Figure 2(a) is plotted for  $g < \gamma_c$ , there are no phonon sidebands. However, spectral peaks of phonon sidebands become visible when  $g > \gamma_c$  [Figs. 2(b) to 2(d)]. Such a condition can be understood by examining the overlap of Lorentzians in Eq. (9). To resolve a peak in the spectrum, the peak height should be higher than the tail of its neighboring Lorentzian. This requires  $g > \gamma_c$  in the resolved sideband regime [14].

From Eq. (10) the peaks of these sidebands are located at  $\Delta_k = (n-m)\omega_M - \delta$ . Therefore there is always a peak at  $\Delta_k = -\delta$  (for  $n = m$ ), which is shown in Figs. 2(b) to 2(d). In particular for  $g < \omega_M$ , the  $\Delta_k = -\delta$  peak appears at the first (labeled from right to left in the re-

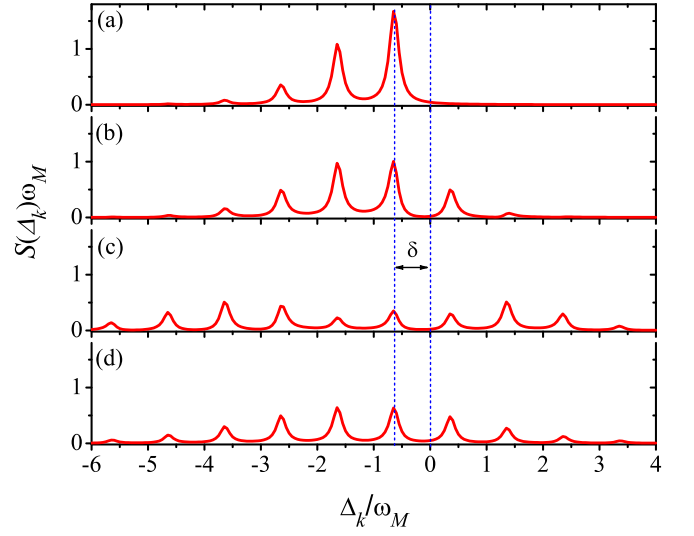


FIG. 3: (Color online) Spectrum  $S(\Delta_k)$  of single-photon emission versus  $\Delta_k$  when the mirror is initially in (a) displaced ground state  $|\tilde{0}\rangle_b$ , (b) ground state  $|0\rangle_b$ , (c) coherent state  $|\beta\rangle_b$  with  $\beta = 3$ , and (d) thermal state  $\rho_b^{th}$  with  $\bar{n}_b = 2$ . Here  $g/\omega_M = 0.8$  and  $\gamma_c/\omega_M = 0.2$ .

gion  $\Delta_k < 0$ ) peak in the red sideband. We remark that the photon-state frequency shift  $\delta = g^2/\omega_M$  can be resolved from the Lorentzian spectrum when  $\delta > \gamma_c$ , which requires  $g > \sqrt{\omega_M \gamma_c}$ .

To illustrate how the spectrum  $S(\Delta_k)$  depends on the mirror's initial state, we plot in Fig. 3 the emission spectrum  $S(\Delta_k)$  as a function of the photon frequency  $\Delta_k$ . Four different initial states are considered here: displaced ground state  $|\tilde{0}\rangle_b$ , ground state  $|0\rangle_b$ , coherent state  $|\beta\rangle_b$ , and thermal state  $\rho_b^{th}$ . When the mirror is initially in  $|\tilde{0}\rangle_b$ , the photon leaves the cavity with a frequency smaller than  $\omega_c$ , and so there are only peaks at the red sideband. In contrast, for the cases of  $|0\rangle_b$ ,  $|\beta\rangle_b$ , and  $\rho_b^{th}$ , the mirror in the displacement representation have some excited-state populations because of  $|n_0\rangle_b = \sum_{\tilde{n}=0}^{\infty} \langle \tilde{n}|_b |n_0\rangle_b |\tilde{n}\rangle_b$ . Consequently there will be some probabilities for the single photon absorbing the phonon's energy and leaving the cavity with a frequency larger than  $\omega_c$ , (i.e., there are some peaks at the blue sideband). Figure 3 also indicates that the number of these peaks depends on the initially contributing phonon distribution in the mirror. The wider the contributing phonon distribution is, the more the peak's number becomes.

*Single-photon scattering.*—Now we turn to the photon scattering problem in which a single photon is injected into the cavity. We assume that initially the cavity is in a vacuum, the single photon in a Lorentzian wave packet [15], and the mirror in number state  $|n_0\rangle_b$ , that is,

$$A_m(0) = 0, \quad B_{m,k}(0) = \sqrt{\frac{\epsilon}{\pi}} \frac{\delta_{m,n_0}}{\Delta_k - \Delta_0 + i\epsilon}, \quad (13)$$

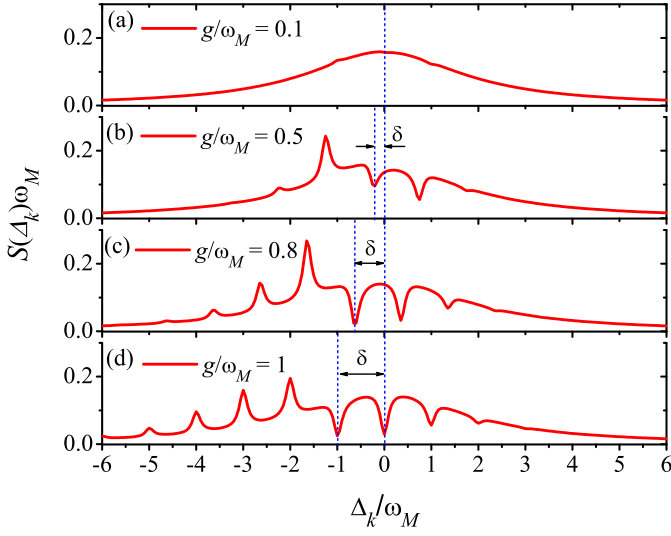


FIG. 4: (Color online) Spectrum  $S(\Delta_k)$  of single-photon scattering versus  $\Delta_k$  for various  $g$ . The initial state of the mirror is  $|0\rangle_b$ . Other parameters are set as  $\gamma_c/\omega_M = 0.2$ ,  $\epsilon/\omega_M = 2$ , and  $\Delta_0/\omega_M = 0$ .

where  $\Delta_0$  and  $\epsilon$  are the detuning and spectral width of the photon, respectively. The scattering solution of the system is obtained as

$$A_{n_0,m}(t) = \sqrt{\frac{\epsilon}{\pi}} \frac{-2\pi\xi\langle\tilde{m}|_b|n_0\rangle_b}{\epsilon - \frac{\gamma_c}{2} + i[(n_0 - m)\omega_M + \Delta_0 + \delta]} \times \left( e^{-[\frac{\gamma_c}{2} + i(m\omega_M - \delta)]t} - e^{-[\epsilon + i(n_0\omega_M + \Delta_0)]t} \right), \quad (14a)$$

$$B_{n_0,m,k}(t) = \sqrt{\frac{\epsilon}{\pi}} \frac{1}{\Delta_k - \Delta_0 + i\epsilon} \delta_{m,n_0} e^{-i(m\omega_M + \Delta_k)t} + \sum_{n=0}^{\infty} \frac{i\gamma_c \sqrt{\epsilon/\pi} \langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b}{\epsilon - \frac{\gamma_c}{2} + i[(n_0 - n)\omega_M + \Delta_0 + \delta]} \times \frac{\left( e^{-i(m\omega_M + \Delta_k)t} - e^{-[\frac{\gamma_c}{2} + i(n\omega_M - \delta)]t} \right)}{\frac{\gamma_c}{2} + i[(n - m)\omega_M - \delta - \Delta_k]} - \sum_{n=0}^{\infty} \frac{i\gamma_c \sqrt{\epsilon/\pi} \langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b}{\epsilon - \frac{\gamma_c}{2} + i[(n_0 - n)\omega_M + \Delta_0 + \delta]} \times \frac{\left( e^{-i(m\omega_M + \Delta_k)t} - e^{-[\epsilon + i(n_0\omega_M + \Delta_0)]t} \right)}{\epsilon + i[(n_0 - m)\omega_M + \Delta_0 - \Delta_k]}. \quad (14b)$$

In the long-time limit,  $A_{n_0,m}(\infty) = 0$  and

$$B_{n_0,m,k}(\infty) = \sqrt{\frac{\epsilon}{\pi}} \frac{e^{-i(m\omega_M + \Delta_k)t}}{\Delta_k - \Delta_0 + i\epsilon} \delta_{m,n_0} - \sqrt{\frac{\epsilon}{\pi}} \frac{i\gamma_c}{\Delta_k - [\Delta_0 + (n_0 - m)\omega_M] + i\epsilon} \times \sum_{n=0}^{\infty} \frac{\langle m|_b|\tilde{n}\rangle_b \langle \tilde{n}|_b|n_0\rangle_b e^{-i(m\omega_M + \Delta_k)t}}{\Delta_k + \delta - (n - m)\omega_M + i\frac{\gamma_c}{2}}. \quad (15)$$

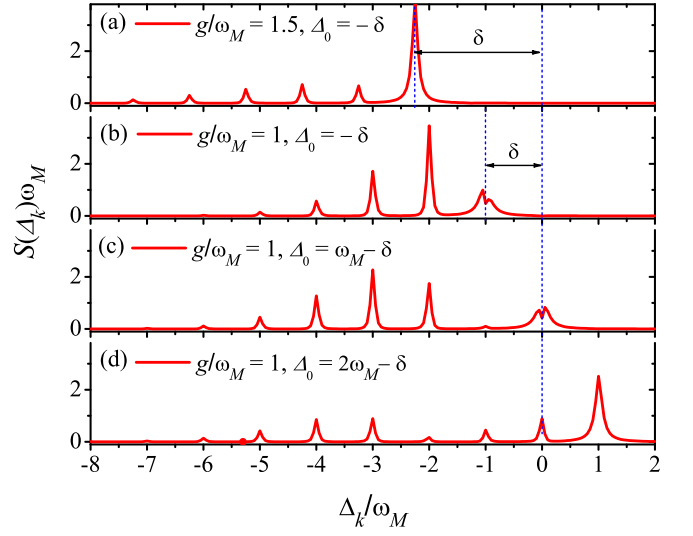


FIG. 5: (Color online) Spectrum  $S(\Delta_k)$  of single-photon scattering versus  $\Delta_k$  for various  $g$  and  $\Delta_0$ . The initial state of the mirror is  $|0\rangle_b$ . Here  $\gamma_c/\omega_m = 0.2$  and  $\epsilon/\omega_M = 0.05$ .

The first term in Eq. (15) corresponds to the process in which the photon is reflected by the fixed cavity mirror without entering the cavity. The second term in Eq. (15) comes from the interaction process after the single photon entering the cavity. Note that the summation part in the second term of Eq. (15) has the same form as that appearing in single-photon emission process discussed above.

Let us first consider the case of  $\epsilon/\omega_M > 1$ . We plot in Fig. 4 the spectrum  $S(\Delta_k)$  of single-photon scattering versus the photon detuning  $\Delta_k$  for various  $g$  when the mirror is initially prepared in ground state  $|0\rangle_b$ . It can be seen from Fig. 4 that phonon sidebands appear in the spectrum when  $g > \gamma_c$ . In addition, the sideband peaks in the spectrum can also be used to characterize the coupling strength  $g$  through the frequency shift  $\delta$ , similar to Fig. 2. It is interesting that these sidebands in the spectrum show both peaks and dips. The reason for these tips is quantum interference between the direct photon reflection channel and the scattering channel relating with the mirror's final state  $|0\rangle_b$ , which are, respectively, the first and second terms in  $B_{0,0,k}(\infty)$  of Eq. (15).

We next consider the case in which the frequency of the incident photon is nearly monochromatic, (i.e.,  $\epsilon \ll \gamma_c$ ). The motivation for studying the case is to see the resonant transition processes by tuning the incident photon frequency  $\Delta_0$ . In Figs. 5(a) and 5(b), we choose  $\Delta_0 = -\delta$  such that the incident photon will excite resonantly the transition  $|0\rangle_a|0\rangle_b \rightarrow |1\rangle_a|\tilde{0}\rangle_b$ . A subsequent cavity photon decay will induce transitions  $|1\rangle_a|\tilde{0}\rangle_b \rightarrow |0\rangle_a|n\rangle_b$  ( $n = 0, 1, 2, \dots$ ). As a result, there are only some peaks at the red sideband, and the first sideband peak measured from  $\Delta_k = 0$  is located at  $\Delta_k = -\delta$ . On the other hand, Figs. 5(b) to 5(d) provide a comparison of the scattering spectrum when the incident photon frequencies are:  $\Delta_0 = -\delta$ ,  $\omega_M - \delta$ , and  $2\omega_M - \delta$  such that the pho-

ton resonantly excites the system from states  $|0\rangle_a|0\rangle_b$  to  $|1\rangle_a|\tilde{0}\rangle_b$ ,  $|1\rangle_a|\tilde{1}\rangle_b$ , and  $|1\rangle_a|\tilde{2}\rangle_b$ , respectively. When the photon leaks out of the cavity, the mirror will transition from  $|\tilde{0}\rangle_b$ ,  $|\tilde{1}\rangle_b$ , and  $|\tilde{2}\rangle_b$  to  $|n\rangle_b$  ( $n = 0, 1, 2, \dots$ ), respectively. Then according to Fig. 1(b) the maximal-frequency sideband peak should be located at  $\Delta_k = -\delta$ ,  $\omega_M - \delta$ , and  $2\omega_M - \delta$ , as illustrated in Fig. 5.

In conclusion, we have calculated analytically the spectrum of single-photon emission and scattering in a cavity

optomechanical system. We have also indicated the connection between the spectral features and the interaction strength  $g$  of radiation pressure per photon. In the resolved sideband regime  $\omega_M \gg \gamma_c$ , the phonon sidebands are visible when  $g > \gamma_c$ , while the condition for resolving the photon-state frequency shift  $\delta$  is  $g > \sqrt{\omega_M \gamma_c}$ .

This work is partially supported by a grant from the Research Grants Council of Hong Kong, Special Administrative Region of China (Project No. CUHK401810).

- 
- [1] T. J. Kippenberg and K. J. Vahala, *Science* **321**, 1172 (2008).
  - [2] F. Marquardt and S. M. Girvin, *Physics* **2**, 40 (2009).
  - [3] S. Gupta, K. L. Moore, K. W. Murch, and D. M. Stamper-Kurn, *Phys. Rev. Lett.* **99**, 213601 (2007).
  - [4] F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, *Science* **322**, 235 (2008).
  - [5] M. Eichenfield, J. Chan, R. M. Camacho, K. J. Vahala, and O. Painter, *Nature (London)* **462**, 78 (2009).
  - [6] S. Mancini, V. I. Manko, and P. Tombesi, *Phys. Rev. A* **55**, 3042 (1997).
  - [7] S. Bose, K. Jacobs, and P. L. Knight, *Phys. Rev. A* **56**, 4175 (1997).
  - [8] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *Phys. Rev. Lett.* **91**, 130401 (2003).
  - [9] P. Rabl, *Phys. Rev. Lett.* **107**, 063601 (2011).
  - [10] A. Nunnenkamp, K. Børkje, and S. M. Girvin, *Phys. Rev. Lett.* **107**, 063602 (2011).
  - [11] T. Hong, H. Yang, H. Miao, and Y. Chen, e-print arXiv:1110.3348.
  - [12] F. A. M. de Oliveira, M. S. Kim, P. L. Knight, and V. Buzek, *Phys. Rev. A* **41**, 2645 (1990).
  - [13] I. E. Linington and B. M. Garraway, *Phys. Rev. A* **77**, 033831 (2008).
  - [14] For example, we consider the case of  $g/\omega_M \ll 1$ . In the resolved sideband regime  $\omega_M \gg \gamma_c$  and under the initial state  $|0\rangle_b$ , the main peak of the spectrum is approximately a Lorentzian function  $S_L(\Delta_k) \approx \frac{\gamma_c}{2\pi} \frac{1}{(\Delta_k + \delta)^2 + (\gamma_c/2)^2}$ . From Eqs. (9) and (12a) the height of the second red sideband can be approximated as  $S(-\omega_M - \delta) \approx \frac{\gamma_c}{2\pi\omega_M^2} \left(1 + \frac{4g^2}{\gamma_c^2}\right)$  up to second order of  $g/\omega_M$ . Then the requirement  $\frac{S(-\omega_M - \delta)}{S_L(-\omega_M - \delta)} \approx 1 + 4g^2/\gamma_c^2 \gg 1$  leads to the condition  $g \gg \gamma_c$ .
  - [15] J. Q. Liao and C. K. Law, *Phys. Rev. A* **82**, 053836 (2010).